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FUEL-CONSUMPTION-MINIMUM-BASED SELECTION OF THE REGIME OF HEATING OF A METAL BY THE MAIN OPTIMIZATION METHOD

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Consideration is given to the solution of the problem of minimization of fuel consumption in heating a rectangular prism in a continuous furnace.

In the case of operation of flame furnaces, problems of minimization of scaling and the decarbonized-layer thickness arise. At present, a decrease in the consumption of a gas over the period of heating of a billet in a furnace is one of the central and most important problems. Problems of this type have been considered extensively in the literature; however, in the majority of works a study has been made of the case not taking into account the temperature distribution over the metal volume. This can lead to the acceptability of the results obtained because of the possible temperature averaging: the maximum or minimum temperature over the cross section will not satisfy the conditions of the technological regime. From this viewpoint, calculation by a two-dimensional scheme is more acceptable.

Let us consider the problem of optimal control by the minimum gas consumption using, as an example, the walking-beam furnace of the 320/150 mill of the Belarusian Metallurgical Plant.

The problem of heating of a prism has been described in [1]. Under the assumptions made in [1, 2], the problem for a prism (Fig. 1) in the case of convective heat exchange has the form

$$\frac{\partial T(x, y, t)}{\partial t} = a^2 \left(\frac{\partial T^2(x, y, t)}{\partial x^2} + \frac{\partial T^2(x, y, t)}{\partial y^2} \right), \quad 0 \le x \le q, \quad 0 \le y \le p, \quad 0 \le t \le t_{\text{fin}}, \tag{1}$$

with initial

$$T(x, y, 0) = T_0(x, y)$$

and boundary conditions

$$\lambda(T) \frac{\partial T(q, y, t)}{\partial x} = \alpha \left(T_{\text{fur}}(t) - T(q, y, t) \right), \quad \frac{\partial T(0, y, t)}{\partial x} = 0; \tag{2}$$

$$\lambda(T) \frac{\partial T(x, p, t)}{\partial y} = \alpha \left(T_{\text{fur}}(t) - T(x, p, t) \right), \quad \frac{\partial T(x, 0, t)}{\partial y} = 0. \tag{3}$$

Since, at the end of heating, the billet must have a temperature distribution over the cross section as close to the prescribed uniform distribution as possible, we have the restriction

$$\max_{\substack{x \in [0,p] \\ y \in [0,q]}} \left| \left(T(x, y, t_{\text{fin}}) - T_{\text{fin}} \right) \right| \le \xi. \tag{4}$$

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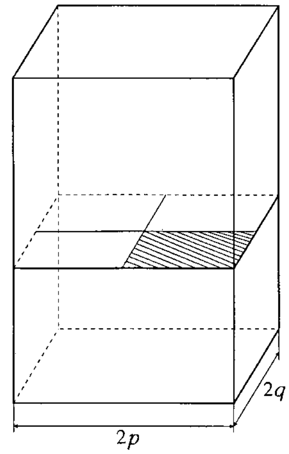

Fig. 1. Diagram of the prism.

We assume [3] that the furnace temperature $T_{\text{fur}}(t)$ is related to the gas consumption $B(t)$ by the following relation:

$$\frac{dT_{\text{fur}}(t)}{dt} = A_1 B(t) - A_2 (T_{\text{fur}}(t) - T_{\text{amb}}) - A_3 \left(\int_0^q \Big[\alpha (T_{\text{fur}}(t) - T(q, y, t)) \Big] dy + \int_0^p \Big[\alpha (T_{\text{fur}}(t) - T(x, p, t)) \Big] dx \right), \qquad (5)$$

where A_1, A_2, and A_3 are certain coefficients [1, 3]:

$$A_1 = \frac{Q_{\text{low}} + V_{\text{a}} C_{\text{a}} T_{\text{a}} + C_{\text{tem}} T_{\text{tem}} - V_{\text{fl}} C_{\text{fl}} T_{\text{out}}}{1.1 M_{\text{tr}} C_{\text{tr}}}, \quad A_2 = \frac{2F_{\text{w}}}{M_{\text{tr}} C_{\text{tr}} (S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{\text{w}})},$$

$$A_3 = \frac{PC_{\text{m}}}{990 \rho C(T) \cdot \text{mes}\, V \cdot M_{\text{tr}} C_{\text{tr}}}.$$

With allowance for the technical restrictions, we write for $t \in [0, t_{\text{fin}}]$ and $T_{\text{fur}}(0) = T^0_{\text{fur}}$

$$B_1 \le B(t) \le B_2, \qquad (6)$$

here B_1 and B_2 are the minimum and maximum consumptions of the gas by the furnace.

The problem of optimal control of heating of a billet by the minimum gas consumption is as follows: it is necessary to find a piecewise-continuous function of the gas consumption $B(t)$, satisfying restriction (6), which would bring the solutions of problem (1)–(5) from the state $T_0(x, y)$ and T^0_{fur} at the instant of time 0 to the state satisfying condition (4), during which the functional minimum could be provided:

$$I(B(t)) = \int_0^{t_{\text{fin}}} B(t)\, dt. \qquad (7)$$

We note that functional (7) characterizes the total gas consumption by the furnace. To solve the problem we use the method of main optimization; in this case, the main operating regime of the furnace is determined as the most useful as far as the increment in the functional is concerned and, obviously, has the form

$$B^*(t) = B_1. \qquad (8)$$

It follows that any solution of problems (1)–(3) and (5), $t \in [t_1, t_2]$, will be the main line on condition that $B(t) = B^*(t)$.

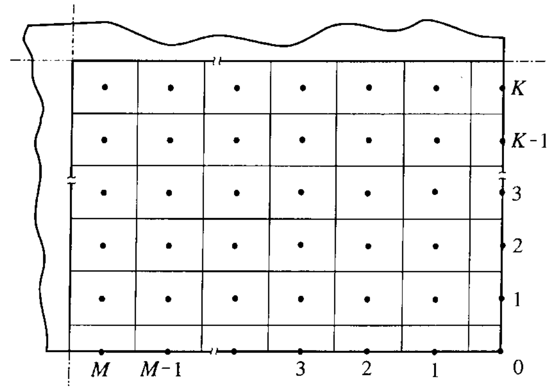

Fig. 2. Diagram of location of the grid region in the cross section of the prism.

From the principle of the maximum it is evident that the optimal regime of heating can take only two values, B_1 and B_2. Therefore, this problem can be characterized by the instants of time of reaching of the main regime by the solution of the temperature problem (in principle, the number of these instants of time can be infinite).

In numerical form, the algorithm can be represented in the following manner: assume that we have a grid (Fig. 2) with nodes (X_m, Y_k, t_n), where

$$X_m = m\Delta x\,, \quad m = \overline{0, M}\,, \quad Y_k = k\Delta y\,, \quad k = \overline{0, K}\,, \quad t_n = n\Delta\tau\,, \quad n = \overline{0, N}\,.$$

Let the grid region fill the prism cross section in such a way that the cross-section boundary passes at the center of the last layers of the grid region

$$T^0_{i,j} = T_0\,(x_i, y_j) \tag{9}$$

for $i \in \overline{0, M}$ and $j \in \overline{0, K}$,

$$T^{n+1}_{i,j} = T^n_{i,j}\left[1 - \frac{2\lambda\Delta\tau}{\rho C}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\right] + \frac{\lambda\Delta\tau}{\rho C}\left(\frac{T^n_{i-1,j}}{\Delta x^2} + \frac{T^n_{i+1,j}}{\Delta x^2} + \frac{T^n_{i,j+1}}{\Delta y^2} + \frac{T^n_{i,j-1}}{\Delta y^2}\right) \tag{10}$$

for $i \in \overline{0, M-1}$ and $j \in \overline{0, K-1}$,

$$T^{n+1}_{i,j} = T^n_{i,j}\left[1 - \frac{2\Delta\tau}{\rho C}\left(\frac{\lambda}{\Delta x^2} + \frac{\lambda}{\Delta y^2} + \frac{\alpha}{\Delta x}\right)\right] + \frac{\Delta\tau}{\rho C}\left(\frac{\lambda T^n_{i-1,j}}{\Delta x^2} + \frac{2\alpha T^n_{\text{fur}}}{\Delta x} + \frac{\lambda T^n_{i,j+1}}{\Delta y^2} + \frac{\lambda T^n_{i,j-1}}{\Delta y^2}\right) \tag{11}$$

for $i \in \overline{0, M-1}$ and $j = K$,

$$T^{n+1}_{i,j} = T^n_{i,j}\left[1 - \frac{2\Delta\tau}{\rho C}\left(\frac{\lambda}{\Delta x^2} + \frac{\lambda}{\Delta y^2} + \frac{\alpha}{\Delta x}\right)\right] + \frac{\Delta\tau}{\rho C}\left(\frac{\lambda t^n_{i-1,j}}{\Delta x^2} + \frac{\lambda t^n_{i+1,j}}{\Delta x^2} + \frac{2\alpha T^n_{\text{fur}}}{\Delta x} + \frac{\lambda t^n_{i,j-1}}{\Delta y^2}\right) \tag{12}$$

for $j \in \overline{0, K-1}$ and $i = M$

$$T^{n+1}_{M,K} = T^n_{M,K}\left[1 - \frac{2\Delta\tau}{\rho C}\left(\frac{\lambda}{\Delta x^2} + \frac{\lambda}{\Delta y^2} + \frac{\alpha}{\Delta x} + \frac{\alpha}{\Delta y}\right)\right] + \frac{\Delta\tau}{\rho C}\left(\frac{\lambda T^n_{M-1,K}}{\Delta x^2} + \frac{2\alpha T^n_{\text{fur}}}{\Delta x} + \frac{2\alpha T^n_{\text{fur}}}{\Delta y} + \frac{\lambda T^n_{M,K-1}}{\Delta y^2}\right), \tag{13}$$

$$T^{n+1}_{\text{fur}} = T^n_{\text{fur}} + \left(A_1 B_n - A_2\,(T^n_{\text{fur}} - T_{\text{amb}}) - A_3\left(\sum_{j=0}^{M}\left[\alpha\,(T^n_{\text{fur}} - T^n_{Kj})\right]\Delta y + \sum_{i=0}^{K}\left[\alpha\,(T^n_{\text{fur}} - T^n_{iM})\right]\Delta x\right)\right)\Delta\tau\,. \tag{14}$$

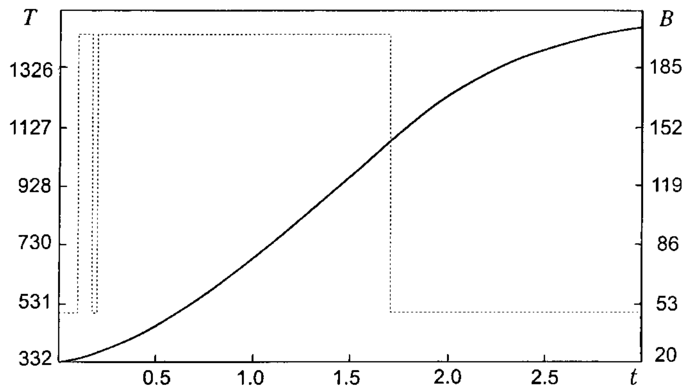

Fig. 3. Change in the temperature of the billet center (solid curve) and in the fuel consumption (dotted curve). T, K; V, m^3; t, h.

All of the above is correct for $n \in \overline{1, N}$.

The functional can be replaced by the following sum:

$$I(B) = \sum_{n=0}^{N} B_n + D \max_{\substack{m=0,M \\ k=0,K}} \left| (T_{mk}^N - T_{\text{fin}}) \right| , \tag{15}$$

where D is a certain constant characterizing a "penalty" for the nonfulfillment of condition (4). In selecting this constant, one must be guided by considerations of the necessity for this condition: at a large constant it is possible to lose the optimal fuel regime, while at a low value of the constant the difference between acceptable and unacceptable solutions will be quite unnoticeable.

This case represents a problem of large-dimensionality mathematical programming. The functional can be minimized according to the following algorithm:

Step 0. Let us assume that the number of instants of reaching of the main regime by the problem solution is $i = 0$ and assign a certain maximum number i^0. Setting the gas consumption for the regime with i instants of reaching to be the maximum possible gas consumption, we have $I(B^0) = B_2 t_{\text{fin}}$.

Step 1. We set $i = i + 1$. If $i > i^0$, we complete the calculations.

Step 2. Next we assign a certain discrete regime for the instants of reaching of the main regime by the problem solution and calculate the fuel consumption $I_0(B^i)$, i.e., functional (15) without accounting for the penalty.

Step 3. Provided that $I(B^{i-1} < I_{\text{with}}(B^i)$, we pass to step 2.

Step 4. We set the instant of time $n = 0$ and determine T_{mk}^n, T_{fur}^n, and B_n.

Step 5. We set $n = n + 1$; if $n > N$, then, before passing to step 2, we calculate functional (15); if the value of the functional is less than the running one, we hold it.

Step 6. Using Eqs. (9)–(15), we calculate T_{mk}^n, T_{fur}^n, and B_n. Then we pass to step 5.

It should be noted that at step 2 the discrete regimes can be generated by the method of exhaustive search for all possible regimes.

The algorithm given in this work was implemented to calculate the optimal regime of metal heating. In the course of the work, we investigated the dependence on the number of acts of reaching the main line. The optimal regime is presented in Fig. 3.

NOTATION

t, running time, h; t_{fin}, time of completion of the heating, h; q and p, half the length and width of a narrow prism face, m; x and y, running coordinates of a narrow prism face, reckoned from the center, m; $T_{\text{fur}}(t)$, furnace temperature at the instant of time t, K; α, coefficient of convective heat exchange, W/(m^2·K); $\lambda(T)$, thermal conductivity, J/(m·h·K); $C(T)$, heat capacity, J/(kg·K); ρ, density, of the material, kg/m^3; $T_0(x, y)$, initial temperature distribution in the prism cross section, K; $T(x, y, t)$, temperature at the point (x, y) at the instant of time t, K; mes$V$, cross-sectional area of the prism, m^2; P, efficiency of the furnace, kg/h; C_m, average specific heat of the metal, J/(kg·K); $B(t)$, fuel consumption, kg/s and m^3/s; V_{fl}, volume of the combustion products formed in burning of 1 kg or 1 m^3 of the fuel, m^3/kg and m^3/m^3; C_{fl}, specific heat of the combustion products, J/(m^3·K); T_{out}, temperature of the outgoing flue gases (taken in conformity with the furnace temperature); T_{amb}, ambient air temperature, K; S_1/λ_1 and S_2/λ_2, thermal resistances (ratios of the thickness of the lining layers to their thermal-conductivity coefficient) for the first and second layers, m^2·K/W; α_w, coefficient of heat transfer from the outer surface of the furnace walls to the environment, W/(m^2·K); F_w, area of the outer surface of the furnace lining, m^2; C_{tr} and M_{tr}, average specific heat (J/(kg·K)) and mass (kg) of the transportation devices found in the furnace during the period Δt; Q_{low}, low combustion heat of the fuel, J/kg and J/m^3; V_a, volume of air needed for burning 1 kg or 1 m^3 of the fuel (with account for the required excess air), J/(m^3·K); T_a, temperature of heating of the air, K; C_f, average specific heat of the fuel, J/(kg·K) and J/(m^3·K); T_f, temperature of the heating of fuel, K; $\xi \geq 0$, certain constant; $T_{fin}(k, y)$, desired (final) temperature distribution in the prism. Subscripts: w, wall; fin, final; tr, transportation devices; fur, furnace; fl, flue gases; out, outgoing flue gases; a, air; m, metal; amb, ambient air; with, without account; low, low.

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